International Journal of Theoretical Physics, Vol. 37, No. 11, 1998

Relativity and the Quantum

Theodore P. Jorgensen[1]

Received June 15, 1998

Beams of entities, such as electrons, may produce diffraction patterns. These patterns may be interpreted in terms of particles and waves. One obvious question concerning these phenomena is, "What is the functional relation between the momentum of the entity and its wavelength?" While this relation is well known, it is of interest to look for another way to arrive at this function using special relativity theory and the fundamental observation that the mathematical form of a law of nature cannot contain any parameters relating to more than one reference frame. It is shown, without making any quantum assumptions, that the relation $P = b/\lambda$, where b is a constant, is valid. This result comes directly from the application of classical nonquantum physics.

The purpose of this paper is to show that the fundamental quantum relationships may be obtained in a more direct fashion with fewer physical assumptions than is usually given in introductory discussions of quantum ideas. Here we shall acknowledge there are entities of interest to physicists which, when formed into beams, may be made to show diffraction effects and countable energy-mass concentrations. When J. J. Thomson studied cathode rays in 1897, he interpreted his experimental results in terms of the particle model with the particles having mass, velocity, and hence energy and momentum. In 1932 his son, G. P. Thomson, studied the same entities and interpreted his observations in terms of the wave model. The entities involved in these studies we call electrons and these investigators were working with electron beams. There are many other entities which, when studied in beams, show energy-mass concentrations and interference effects similar to those shown by electrons. It is unfortunate that physics has no accepted generic term for these entities.

[1] Physics Department, University of Nebraska, Lincoln, 68588.

We shall continue using the wave and particle models, acknowledging that special relativity theory (Moller, 1952) was developed in terms of these intuitive models. We have no other models to use in discussing these entities. We should not be surprised if their use leads us to quantities, such as phase velocity, which cannot be measured in the laboratory.

In the case of the electron beam experiments, both the momentum P of the electrons and the wavelength λ vary with the potential difference across the electron gun. We ask the general question for all these entities, "How is the momentum of the entity related to its wavelength?" We are searching for a functional relation between the momentum P of the entity and its wavelength λ which possibly may be considered to be a "law of nature." Since the wave vector K, which has the magnitude $1/\lambda$, is more convenient to use, we shall seek the functional relation $P = f(K)$.

For a given entity, a diffraction pattern can be obtained only with the accumulation of a large number of energy-mass concentrations. Experimental work has shown that the diffraction pattern obtained with a "weak beam," comprising only one entity, if localized within the diffraction apparatus at one time, is the same as the pattern obtained with an intense beam. It appears that interference occurs with an individual entity.

Of course we know the functional relation we are seeking. De Broglie (1924) made the happy guess that Einstein's (1905) heuristic relation $E = h\nu$, in the modern notation, applied to mass particles as well as to photons, and obtained the relation $P = h/\lambda$. This discussion is based on the fact that mass-energy concentrations and diffraction patterns are characteristics of beams of these entities and not on any precise experimental measurements.

We shall use the Lorentz transformation equations of the energy-momentum and wave four-vectors. In the notation we shall use, these vectors have the components P_x, P_y, P_x, iE/c and K_x, K_y, K_z, $i\nu/c$.

Consider a laboratory in a particular reference frame S_0 in which all relevant measurements on entities of a particular kind may be made. Consider a similar laboratory in *any* other reference frame S moving away from the source of these entities with a constant velocity w relative to reference frame S_0. We shall choose our x axis in the direction of w. The transformation equations of special relativity may be used to calculate the momentum P and the wave vector K of an entity in the reference frame S in terms of the corresponding quantities P_0 and K_0 in the reference frame S_0. The Lorentz spacelike transformations give $P = \gamma(P_0 - wE_0/c^2)$ and $K = \gamma(K_0 - w\nu_0/c^2)$, where E_0 and ν_0 are the energy and the frequency, respectively, in the reference frame S_0 and where the quantity γ is given by $\gamma = 1/\sqrt{1 - (w/c)^2}$.

We shall use v_0 to represent the particle velocity and u_0 to represent the wave phase velocity in the reference frame S_0. The vectors P, P_0, K, K_0, v_0,

u_0, and w are collinear. Thus we shall consider the problem in one dimension; a three-dimensional approach is found to give the same result. Using $E = mc^2$, $\lambda\nu = u$, $K\lambda = 1$, and $P = mv$, we find $P = \gamma P_0(1 - w/v_0)$ and $K = \gamma K_0(1 - wu_0/c^2)$.

If these two four-vectors are functionally related, that is, if there is a relation $P = f(K)$, it should be possible to use these two equations to find information about this relation. Before we may accept any mathematical expression as a statement of a law of nature, we must consider the criteria which a particular expression must satisfy before it may be accepted as such. This topic has been critically considered by Bridgman (1962), who gave two criteria which will be helpful in determining whether any functional relation can be an expression of a law of nature. His conclusions are, if paraphrased correctly, that such a mathematical expression may contain only parameters associated with a particular frame of reference, here reference frame S, and must be shown to be valid when checked with experiment. Therefore an expression $P = f(K)$ cannot also be a function of w.

Let us divide the last two equations in order to remove γ from our consideration. The resulting equation $(P/K) = (P_0/K_0)(1 - w/v_0)(1 - wu_0/c^2)^{-1}$ is valid for all values of w. When $wu_0/c^2 < 1$ the last factor may be expanded as a power series of w. This equation becomes

$$P/K = (P_0/K_0)\{1 + (u_0/c^2 - 1/v_0)[w + (u_0/c^2)w^2 + (u_0/c^2)^2 w^3 + \ldots]\}$$

This equation is of the form $P/K = A_0 + A_1 w + A_2 w^2 + A_3 w^3 + \ldots$. But if there is a law of nature expressed by this equation, then this equation must have all of the A's except A_0 equal to zero. We see that $A_1 = (P_0/K_0)(u_0/c^2 - 1/v_0)$, and if this is equal to zero, then we find that $u_0 v_0 = c^2$. It may be shown that this expression is relativistic invariant. We may take Bridgman at his word and arbitrarily remove all the terms involving w from the equation or we may observe that since $u_0 v_0 = c^2$ all the terms involving w disappear from the equation. Either way, we find $P = K(P_0/K_0)$ and since P_0/K_0 is the ratio of two measurements made on the entity in the laboratory at rest in frame S_0, this ratio is a constant. We shall call it b.

This example shown here in detail deals with the space components of the four-vectors of the entity. When a similar procedure is applied to the time components of the same four-vectors, we find again that $u_0 v_0 = c^2$ and that $E = b'\nu$, where b' was obtained in the same manner as was b. But since, for the space and the time components, $u_0 v_0 = c^2$, we may use this fact to show that b and b' are equal. We see that now $b' = E/\nu = mc^2\lambda/\lambda\nu = mvc^2\lambda/uv = P\lambda c^2/uv = P\lambda = b$ or $b' = b$.

Since the relation $P = bK$ is true for any frame of reference S, we find that $P = b/\lambda$ is the same for all frames and is thus relativistic invariant. It also follows that $E = b\nu$ is also relativistic invariant. The second criterion of Bridgman is shown to be satisfied by the development of physics in the past 100 years.

It appears that we have reached the goal stated earlier. We have found, using special relativity theory and making no quantum assumptions, while recognizing that b and b' are equal, that $P = b/\lambda$ and $E = b\nu$. All experimental determinations of b have shown no indication that the value of this constant is different for different entities. Of course this constant is Planck's constant It appears that the wave phenomena we observe in the laboratory are associated with these entities through interference.

ACKNOWLEDGMENTS

I wish to acknowledge the criticism and encouragement over the years of colleagues Timothy J. Gay, Sitaram S. Jaswal, Joseph H. Macek, Edgar A. Pearlstein, Anthony F. Starace, and Henry S. Valk.

REFERENCES

Bridgman, P. W. (1962). *A Sophisticate's Primer of Relativity*, Wesleyan University Press, Middletown, Connecticut, p. 118.

de Broglie, L. (1924). *Philosophical Magazine*, **47**, 446.

Einstein, A. (1905). *Annalen der Physik*, **17**, 132.

Moller, C. (1952). *The Theory of Relativity*, Oxford University Press, Oxford.

Taylor, G. I. (1909). *Proceedings of the Cambridge Philosophical Society*, **15**, 114.

Thomson, G. P., and Cochrane, W. (1939). *Theory and Practice of Electron Diffraction*, Macmillan, London, p. 224.